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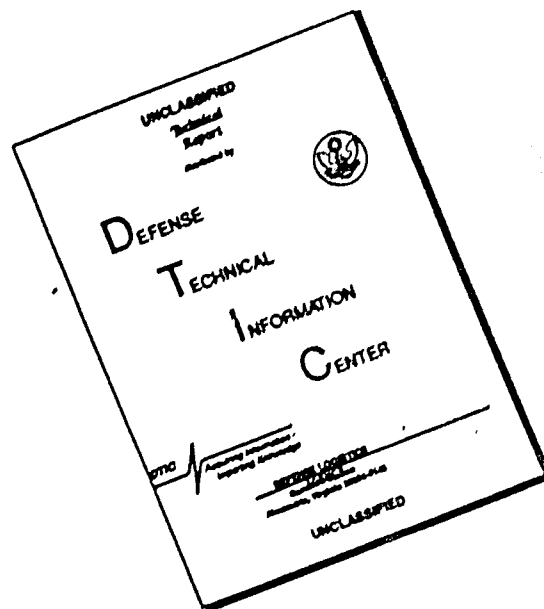
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## SPACECRAFT MULTI-BEAM AND CONTOURED-BEAM ANTENNAS

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## SUMMARY

High-gain spacecraft antennas with multiple beams and contoured beams are key components in satellite communications and direct broadcast systems. This is reflected on the latest generation of communications satellites, where the antenna subsystem is the largest subsystem with its weight of more than 300 kg. The antennas achieve a large communications capacity through multiple frequency re-uses and may be reconfigured to serve different coverage areas.

The paper overviews the current multi-beam and contoured-beam antenna technology. Different implementations, reflector or lens with feed array or direct radiating array, are considered. The emphasis is placed upon systems with offset paraboloidal reflectors. The limitations of the offset reflector with respect to beam scanning and cross polarization are reviewed. Computer-aided design procedures and design examples are presented.

## 1. INTRODUCTION

The antenna systems carried on board spacecraft have over the last 25 years undergone a rapid evolution which parallels that of the spacecraft themselves. In fact, the spacecraft system requirements have been and continue to be a main driving force behind the rejuvenation of antenna theory and technology which previously were considered to be mature disciplines. This evolution has been supported by the simultaneous advent of abundant computational facilities.

Early spacecraft were small, spin-stabilized satellites in low orbits. The antenna systems were simple, often with a low power in a narrow frequency band, typically in the VHF band. Current spacecraft have become large and highly specialized and often carry several antenna systems which are tailored to the role of the spacecraft. Most communications spacecraft are placed along the geostationary arc some 36,000 km above the surface of the Earth. The multi-beam and the contoured-beam antennas reviewed in this paper are examples of particularly complex antenna systems which significantly increase the capacity and flexibility of these satellite systems. By international agreement certain frequency bands have been allocated for the different satellite services and rules have been set to minimize the interference with other satellite systems and with earth-based systems [1]. Typically, the uplink signals from the Earth to the satellite and the downlink signals back to the Earth use different bands. Some of the most important frequency-band allocations for the fixed satellite services (communications satellites) are indicated in Table 1. The frequency allocations differ slightly for three CCIR regions of the world and various restrictions may apply so that the full bands cannot be used. Other bands are allocated for communication with mobile stations and for direct broadcast. Additional bands are allocated at higher frequencies. Initially, the lower bands have been the most popular as the technology has been better developed.

	Uplink	Downlink
C-band	5.925- 7.250 GHz	3.400- 4.200 and 4.500- 4.800 GHz
Ku-band	12.500-13.250 and 14.000-14.800 GHz	10.700-11.700 GHz
Ka-band	27.500-31.000 GHz	18.100-21.200 GHz

Table 1. CCIR allocations for the fixed-satellite service

## 1.1 Multi-Beam Antennas

The finite frequency spectrum available and the finite number of slots along the geostationary arc for satellites operating in the same frequency band are best utilized by a multi-beam antenna which illuminates the coverage area by a number of element beams. A hexagonal beam lattice is the most efficient for contiguous area coverage. If the antenna radiates  $N$  beams and the available frequency spectrum is divided into  $K$  bands or channels so that adjacent beams use different bands, the frequency spectrum can be re-used  $N \times K$  times. Figure 1 shows different beam topologies where the number in each cell or beam foot print gives the channel number. The more channels the frequency band is divided into, the larger the spacing will be between the cells where the same frequency is re-used and the better the isolation will be between these beams. However, the number of times the frequency is re-used will be less. In practical systems, adjacent or overlapping beams may use orthogonal polarizations to improve the isolation or increase the number of frequency re-uses.

As an additional advantage of dividing the coverage area into smaller cells, the spacecraft antenna gain is drastically increased. If all the power radiated by the satellite were uniformly distributed over its field-of-view subtended by the Earth, the ultimately achievable gain would be 22.4 dBi. This would require an infinitely large, lossless antenna. In practice less than 17 dBi is obtained towards the edge of the Earth using a horn antenna. With a multi-beam antenna, the upper limit on the achievable gain would be set by size of the element-beam foot print or by the acceptable size and complexity of the antenna system and the accuracy with which the antenna can be kept pointed towards the coverage area. This shift

from the early low-gain over medium-gain spacecraft antennas, which required huge earth-station antennas, to high-gain spacecraft antennas with small beam foot prints has in conjunction with improvements in satellite power and low-noise receivers lead to the introduction of comparatively cheap, small earth-station antennas. More aspects of multi-beam antennas are discussed in [2-4].

The simple beam topologies of Figures 1 carry some disadvantages. In real life, the communications requirements are not uniformly distributed over the field-of-view and a multi-beam antenna system with many beams requires many transponders and a large switch matrix to provide the inter-connectivity between the uplink and the downlink beams. Also, it has not yet been practical without the use of several antennas or excessive antenna losses to radiate element beams with the crossover levels down from the beam peak in the order of 3-4 dB needed for a contiguous coverage. These problems have led to the concept of contoured-beam antennas where several of the element beams are combined in a cluster or a composite beam.

### 1.2 Contoured-Beam Antennas

A contoured-beam antenna provides one or more beams with foot prints on the Earth tailored to specific geographical areas. Sometimes these antennas are referred to as multi-beam antennas or shaped-beam antennas. The beam-contouring conserves the satellite power and reduces the interference both with adjacent frequency re-use coverage areas of the same satellite system and with other systems. Figure 2 illustrates the most common hardware used to generate a contoured beam: a feed array which illuminates an offset paraboloidal reflector. Each feed element, usually a small horn, generates a scanned pencil beam which is termed an element beam or a component beam. The foot prints of these element beams on the Earth are indicated by small circles on Figure 2. A contoured beam which provides service to coverage area A is obtained by adding the element beams radiated towards the coverage area with appropriate (complex) weight factors. These weights or feed excitations are generated by a beam-forming network (BFN) which often is a power divider tree with phase shifts provided by line length differences at the feed ports.

The feed array and in particular the BFN are the most critical parts of a contoured-beam reflector antenna as they must realize the desired feed excitations with acceptable amplitude and phase tolerances and low VSWRs at both the feed ports and the beam port over the operating frequency bandwidth. The antenna system is required to operate in a hostile space environment with temperature excursions in the order of -60 to 60 °C or more over the 7 to 10 year satellite lifetime. The BFNs of most current contoured-beam antennas consist of fixed power dividers and phase shifters. These BFNs are usually implemented in TEM-line in the 6/4 GHz bands and in waveguide in the 14/11 and 30/20 GHz bands. Waveguide BFNs have lower insertion loss, but are heavier and more bulky than TEM-line BFNs. The stringent matching requirements have led to the almost exclusive use of hybrid couplers rather than simple 3-port TEEs or E-plane couplers in the power divider tree.

Advanced antenna systems include on international communications satellites switches to provide slowly reconfigurable beams to allow a satellite to operate from different locations along the geostationary arc and to accommodate traffic changes and on DSCS III fully reconfigurable BFNs with variable ferrite power dividers. Future systems are likely to include more variable power dividers and variable phase shifters in ferrite or solid-state technology. This will allow both a high degree of beam flexibility and the fast reconfigurability required for hopping and scanning beams with TDMA. Ultimately, BFNs are expected to include many active components to compensate for losses. This will provide very compact and flexible BFNs.

If the antenna system only is required to generate a single fixed contoured beam, a shaped reflector illuminated by a single feed is an attractive solution with respect to both performance and cost mainly because no BFN is required. This concept is reviewed in Section 5.

### 1.3 Choice of Reflector, Lens or Array?

The use of a focusing device such as a reflector or a lens provides a one-to-one relation between the element beams and the feed horns. This relationship does not exist in the case of an array, where each array element contributes to all element beams, and an array with several independent beams operating at the same frequency and polarization would suffer from significant losses unless a Butler-matrix type BFN is used with orthogonal array illuminations for the different beams. In the case of multi-beam antennas where each beam can be generated by a single feed and of contoured-beam antennas where only a few element beams are used to generate the contoured beam, the BFN is much simpler for a reflector and a lens. As a result, array antennas have found only little use as spacecraft multi-beam and contoured-beam antennas. Nevertheless, we will consider contoured-beam array antennas further in Section 7. A review of multi-beam arrays is given in [5].

The choice between reflector and lens is more difficult. In the past, most systems have used reflectors due to their low weight and cost, excellent bandwidth and polarization properties and the fact that they can be analysed very accurately with the existing RF analysis methods. A main disadvantage of the reflector is the need to use an offset-fed reflector geometry to avoid blockage by the large feed array, BFN and support structures. The offset-fed reflector has significantly worse scan and cross-polarisation performance and occupies a larger volume on the spacecraft than a similar center-fed reflector.

The lens is a focusing device with axial symmetry but without blockage. As furthermore the remaining dominant scan aberration, coma, may be removed by choosing the inner lens surface to be a sphere so that the Abbe sine condition is fulfilled, lens antennas have considerable attraction for multi-beam applications [6]. Dielectric lenses are far too heavy for use at microwave frequencies on a spacecraft, and new low-weight but also more complicated lenses such as the soned waveguide lens and the printed-circuit bootlace or TEM-line lens outlined in Figure 3 have been devised. These lenses may be attractive in systems with somewhat less than 100 element beams where the number of waveguide or printed-circuit elements can be kept reasonably small. The DSCS III spacecraft in fact flies three multi-beam soned waveguide lens antennas operating at about 8 GHz [4]. The bandwidth of a soned waveguide

lens is not compatible with those stated in Table 1. The more broadbanded TEM-line lens has been investigated for use at C band [7]. However, its weight was considered to be so large that the receive and the transmit function would have to be combined in one antenna. The matching problem at the inner lens surface could not be solved over the combined frequency band and led to degradations of the element beams near the axis. Because of their complexity and the still unsolved problems, lens antennas are not considered further in this paper. It is expected that lens antennas will prove to be more useful at millimeter and submillimeter wavelengths.

Solid reflectors manufactured from carbon-fiber re-reinforced plastic (CFRP) have become very popular due to their low weight and excellent thermal behavior. Surface accuracies in the order of  $1/100$  wavelength RMS are required to ensure low sidelobes. Solid offset reflectors provide excellent cross-polarization performance when used with circular polarization. However, many domestic systems use linear polarization. This has led to the development of gridded and dual gridded reflectors to reduce the cross polarization introduced by the offset configuration.

## 2. FUNDAMENTALS OF CONTOURED-BEAM REFLECTOR ANTENNAS

### 2.1. Basic Definitions

Contoured-beam reflector antenna systems have a unique set of performance parameters. Most other antenna systems, including earth station antennas, optimize the on-axis gain subject to certain sidelobe constraints often defined by an envelope. The driving system parameters are EIRP and G/T with sidelobe constraints added to minimize interference. Cross-polarization requirements often only apply near the beam axis. When an antenna is required to serve an area rather than just a single direction, the minimum coverage area gain and not the peak gain becomes the significant parameter. Hence, the standard definition of antenna efficiency does not apply to contoured-beam antennas. The efficiency  $\eta$  of a contoured beam antenna is defined as the ratio of the minimum coverage area gain MCAG to the gain  $G_{un}$  of a lossless antenna which distributes all the radiated power uniformly across the specified coverage area  $\Omega$  (in steradians), i.e.,

$$\eta = MCAG/G_{un} \quad (1)$$

where

$$G_{un} = 4\pi/\Omega. \quad (2)$$

Alternatively, the gain-area product,  $MCAG \times \Omega$ , may be defined. Due to the finite satellites pointing accuracy, the gain slope must be controlled within the area of uncertainty for each earth station. The area of uncertainty is called the pointing-error box, sphere or ellipse dependent upon its actual shape which is determined by the satellite attitude control system. The coverage area  $\Omega$  must include the pointing error.

A pencil beam with a circular foot print is the simplest example of a beam with an area coverage. If we assume a Gaussian beam shape and no losses, the power pattern

$$G(\theta) = 4 \ln 10 / \theta_{10}^2 \cdot 10^{-(\theta/\theta_{10})^2} \quad (3)$$

gives the directivity. The polar angle  $\theta$  is measured from the beam axis and  $\theta_{10}$  is half the 10-dB beam-width. The minimum coverage area gain occurs at the edge of the coverage area,  $\theta = \theta_c$ , and is maximized for  $\theta_{10} = \theta_c / \ln 10$ . This result also applies for Gaussian beams with elliptical foot prints. Thus, the minimum coverage area gain is about 4.3 dB below peak gain for circular and elliptical foot prints. The associated gain-area product is  $4\pi \cdot 10^{-\ln 10}$  or  $15176 \text{ degrees}^2$ , which corresponds to a contoured-beam efficiency of only 36.8 per cent even though that all losses have been neglected. This efficiency, which accounts for the great gain difference between the ideal and the practical global-coverage antenna in Section 1.1, may be improved by a flatter gain over the coverage area and a steeper gain fall off at the edge of the coverage area.

When the antenna system provides multiple contoured beams, the driving antenna parameter becomes the isolation which can be achieved between beams which re-use the same frequency band either through spatially separated copolarized beams or through orthogonally polarized beams. Figure 4 illustrates the different definitions of inter-beam isolation which apply for receive and transmit satellite antennas. When the satellite antenna transmits, an earth station served by beam B may receive an interfering signal via a sidelobe of beam A. The transmit isolation is defined as the ratio of the desired signal from beam B to the undesired signal from beam A and depends on both the relative antenna gain and the relative transmit power, i.e., the relative EIRP. With the satellite antenna receiving, the interference occurs via the sidelobes of the beam itself from earth stations outside the service area. If all earth stations transmit with the same EIRP, the receive isolation is defined as the ratio of the antenna gain towards the earth station in the coverage area to the antenna gain in the direction of the interfering earth station at the highest sidelobe level. The receive isolation is considered to be the most difficult as a single high-level sidelobe which falls in a frequency re-use coverage, will destroy the isolation on all coverage-area stations.

The minimum spacing between adjacent copolarized frequency re-use coverages, measured in beamwidths of the element beams, determines the required aperture size or resolution of the antenna system. An antenna pointing error of  $\epsilon$  will reduce the minimum coverage area spacing by  $2\epsilon$ . Thus, a large pointing error may require a significantly larger and more complex antenna system.

## 2.2. Overview of the Current State-of-the-Art

The INTELSAT communications satellites provide an important example of the practical use of reflector antenna systems with multiple contoured beams. The complexity of in particular the C-band antenna systems have grown significant for each new spacecraft series as illustrated by the increasing complexity of the coverages shown in Figure 5. INTELSAT IV A introduced two-fold frequency re-use through two spatially isolated hemispherical beams by means of an array-fed offset paraboloidal reflector antenna system [8]. This concept was further developed on INTELSAT V where four-fold frequency re-use was realized by the addition of two smaller zone beams in the opposite sense of circular polarization. These zone beams have one shape when the spacecraft operates over the Atlantic or the Pacific Ocean region and another over the Indian Ocean region [9, 10]. The trends towards both more frequency re-uses and more reconfigurability of the beam coverage contours continue on INTELSAT VI. Two hemispherical beams and four zone beams in opposite senses of circular polarization provide six frequency re-uses. The four zone beams are reconfigured for each of the three ocean regions providing a total of 14 coverage beams with six active in a given ocean region [11, 12]. The achievements and the limitations of this technology are summarized in Table 2 [13]. It is common practice to use separate antennas for the transmit and receive function to reduce both the bandwidth over which an antenna is required to operate and problems associated with passive intermodulation.

The increased number of frequency re-uses sets stringent sidelobe and cross-polarization requirements in the order of 27-30 dB over a field-of-view which extends approximately  $\pm 10^\circ$  from the subearth point to accommodate both antenna pointing errors and the spacecraft pitch biasing used to maximize the minimum spacing between spatially isolated beams. As the spacecraft are placed above the middle of the oceans, the beam coverages fall near the maximum scan angle so that scan aberrations degrade the achievable sidelobe isolation. So far, however, the diameter of the spacecraft antenna, measured in wavelengths, has been moderate, and the number of beamwidths across less than 6-7 half-power beamwidths. This is demonstrated by the observation that for all satellites in the INTELSAT IVA, V to VI series the minimum separation between two copolarized coverages is about 1.5 element-beam beamwidths for 27 dB isolation [14]. This relationship may continue for significantly larger reflector diameters if longer  $f/D$  ratios or more complex feed arrays and RFNs can be accommodated to reduce the aberrations of the scanned element beams. Figure 6 shows the calculated contoured-beam efficiency versus coverage area size for INTELSAT VI. The two upper curves apply for a smaller feed element diameter of about  $1.3 \lambda$  and in case of the uppermost curve also the effect of a slightly increased spacing between the coverage areas [15]. Other contoured-beam antennas also for domestic/regional and direct broadcast systems are reviewed in [16-20].

## 2.3. Analytic Model of Reflector Element Beams

A simple analytic model is presented for the element beams radiated by small circular or square feeds in an offset paraboloidal reflector with circular aperture. Effects due to the illumination taper and the spillover are included to provide an accurate assessment of the achievable minimum coverage area gain. The model neglects scan aberrations and cross-polarization and is therefore best suited for reasonably large  $f/D$  ratios and/or small scan angles. Even then we have found that the model may give surprisingly good predictions for the minimum coverage area gain and the average sidelobe level. The model cannot be used to determine the number of feeds or the feed excitations accurately. Therefore, the model is most useful in initial trade-off studies to determine approximately the antenna size and feed complexity given the coverage specifications. The detailed design optimization should be carried out using element beams calculated by an accurate reflector antenna analysis program such as GRASP [21].

Let the aperture distribution due to a single feed be approximated by

$$g(\rho) = \alpha_0 + (1 - \alpha_0) [1 - (\rho/a)^2]^n, \quad (4)$$

where the radial variable  $\rho < a$ . The parameter  $\alpha_0$  is the relative illumination at the reflector edge. Typical parameter values corresponding to an element beam would be  $n = 1$  and  $\alpha_0 = 0.7$ . We assume that this amplitude distribution applies for all element beams. As we neglect scan aberrations, the phase distributions caused by the lateral displacement of the feeds in the tilted focal plane only direct the wave fronts towards the positions in the beam grid. The element beams are approximated by the normalized patterns

$$E_j(\theta, \phi) = ke [\alpha A_1(kex_j) + \beta A_{n+1}(kex_j)]. \quad (5)$$

The functions  $A_n(x)$  are given by Bessel functions  $A_n(x) = 2^{n+1} (n+1)! J_{n+1}(x)/x^{n+1}$  so that  $A_n(0) = 1$ . The argument depends on the perimeter  $ke$  of the circular aperture in wavelengths and the distance

$$x_j = ((u-u_j)^2 + (v-v_j)^2)^{1/2} \quad (6)$$

between the field direction  $(u, v)$  and the beam direction  $(u_j, v_j)$  in the  $uv$ -plane where  $u = \sin\theta \cos\phi$  and  $v = \sin\theta \sin\phi$ . The angles  $\theta$  and  $\phi$  are the polar and the azimuthal angle in a standard spherical coordinate system directed along the antenna boresight. The pattern parameters  $\alpha$  and  $\beta$  in (5) depend via the edge taper  $\alpha_0$  upon the primary parameters

$$\begin{aligned} \Delta\theta_3 &- \text{the half-power element beam beamwidth, and} \\ \theta_B &- \text{the element beam spacing} \end{aligned}$$

as outlined below. In order to model the offset reflector and the feed element including spillover losses, the following secondary parameters

$$\begin{aligned} d_o/D &- \text{the relative offset height or clearance,} \\ d_e &- \text{the feed element spacing in wavelengths, and} \end{aligned}$$

$\theta_{10}$  - half the 10-dB feed beamwidth

ere specified. These parameters are indicated on Figure 7. Given the above primary and secondary parameters, the parabola focal length  $f$  and the reflector diameter  $D$  can be derived. The reflector clearance  $d_c$ , the distance from the parabola axis to the reflector edge, should be so large that no scanned beams are blocked by the feed array. The feed element spacing  $d_e$  and the feed beamwidth  $2\theta_{10}$  are roughly inversely proportional for a given feed type.

Once the above parameters are given, an initial values of the reflector diameter  $D$ , the focal length  $f$  and subtended semi angle  $\theta^*$  are calculated assuming a 3 dB edge taper by the approximate formulae:

$$D = 1.029 (1 - 0.212 \log_{10} \alpha_0) / \Delta \theta_3 \quad (7)$$

$$d_1 = (0.5 + d_c/D) D \quad (8)$$

$$f = (K d_e / \theta_B + \sqrt{(K d_e / \theta_B)^2 - d_1^2}) / 2 \quad (9)$$

$$\theta^* = \text{Arctg } d_1/f - \text{Arctg } d_c/2f \quad (10)$$

where  $K = 0.97$ . All angles are in radians and all lengths in wavelengths. Note that (9) has no solution if  $\theta_B > K d_e / d_1$ . An improved value may now be calculated for the average aperture edge illumination using

$$\alpha_0 = 10^{-(\theta^*/\theta_{10})^2} / (1 + D^2/8(f^2 + D^2)) \quad (11)$$

Given this value, the values for  $D$ ,  $d_1$ ,  $f$  and  $\theta^*$  are updated using (7) through (10). The offset paraboloid is now completely determined, and the pattern coefficients are determined from

$$\alpha = \alpha_0 / N L_{so} \quad (12)$$

$$\beta = (1 - \alpha_0) / [(1+n)N] L_{so} \quad (13)$$

$$N = [(1+n+2n\alpha_0+2n^2\alpha_0^2)/(1+3n+2n^2)]^{1/2} \quad (14)$$

where

$$L_{so} = 1 - 10^{-(\theta^*/\theta_{10})^2} \quad (15)$$

gives the element-beam spillover loss. The pattern coefficients are normalized so that the pattern (5) squared gives the directivity. This normalization is convenient for the normalization of the feed excitations discussed in Section 2.5. The effect of typical element-beam edge taper on the directivity is very small, about 0.1 dB or less. The corresponding element-beam spillover loss is more significant as discussed in Section 2.7. A number of other beam models or simple design rules are available for initial contoured-beam reflector antenna trade-off and layout [22, 23].

## 2.4 Array Elements

The array element is a key element in determining the overall performance. The feed element must be chosen in accordance with the reflector geometry. For a specified beam-spacing/beamwidth ratio,  $\theta_B/\Delta\theta_3$ , a smaller feed requires a shorter  $f/D$  ratio of the parabola than a larger feed. The  $\theta_B/\Delta\theta_3$  ratio is usually close to unity corresponding to an element-beam crossover level of about -3 dB. The feed diameters in the range from 1 to 1.6  $\lambda$  match well  $f/D$  ratios in the range from 0.7 to 1.4. For small  $f/D$  ratios, the reflector subtended angle becomes larger and the spillover losses smaller. However, the scan losses increase. The spillover losses are minimized if the product  $\theta_{10} d_e$  is kept small. This parameter plays the role of an array-element quality number and is relatively independent of the size of a particular type of element as already discussed in Section 2.3. For a small circular fundamental-mode horn the value of this parameter is about 1.00 while it is about 1.17 for a small dual-mode or Potter horn. Small corrugated horns are quite useless as array elements because of the large space taken up by the corrugations. From these considerations it would appear that small fundamental mode horn would be the most useful for contoured-beam antenna applications. However, the mutual coupling in the array environment will in practice annihilate the excellent theoretical performance of the small fundamental-mode radiator.

The circular waveguide feed elements discussed above and the square waveguide feed elements are used commonly in circularly polarized antenna systems. Many linearly polarized domestic/regional systems use rectangular feeds of different sizes with or without dielectric loading often illuminating a dual gridded plane of the reflector. These systems may realize very respectable values of the minimum coverage area gain with a minimum of antenna hardware but are not dealt with in the paper [24, 25].

In a dual circularly polarized antenna, a feed element in conventional waveguide technology consists of a horn radiator, a polarizer and an orthomode transducer. In a large antenna system with many feeds, this may be a very bulky and mechanically fragile system. Therefore, microstrip patch radiators have recently received considerable attention as a potential very compact replacement.

## 2.5 Network Loss and Feed Excitation Normalization

When several feeds are excited simultaneously to generate a contoured beam, the overall edge illumination and spillover loss will in general decrease. On the other hand, the beam forming network (BFN) required to generate the optimized amplitude and phase distribution at the feed ports introduces Ohmic and mismatch losses. This BFN loss increases with the number of feed ports as more layers of power dividers and longer line lengths are required in the power divider tree. Fully reconfigurable BFNs may have significantly higher losses than fixed BFNs.

The feed excitations  $e_j$  must be normalized by

$$P_{in} = 4\pi \sum_{j=1}^N |a_j|^2 = \begin{cases} 4\pi & \text{for directivity} \\ 4\pi 10^{-LBFN/10} & \text{for gain.} \end{cases} \quad (16)$$

Then, the directivity or the gain will be referred to the total incident power at either the feed apertures or at the BFN input port. The directivity referred to the total radiated power is derived if the excitation normalization is

$$P_{rad} = 4\pi \sum_{i=1}^N \sum_{j=1}^N r_{ij} e_i a_j^* = 4\pi, \quad (17)$$

where  $r_{ij}$  is the normalized mutual resistance and  $r_{ii} = 1$ .

## 2.6. Mutual Coupling

For a sufficiently large array element with a Gaussian pattern, the normalized mutual resistance is

$$r_{12} = e^{-(kd_{12}\theta_{10})^2/4\ln 10}. \quad (18)$$

This indicates that the mutual resistance depends with the approximations made only on the product  $d_{12}\theta_{10}$ , which for adjacent feeds is equal to the feed quality number. Thus, the requirements for low mutual coupling and for a small spillover loss are in conflict: A large value of  $d_{12}\theta_{10}$  reduces the mutual coupling but increases the element beam spillover loss. In a study of feed-array directivity, cosine approximations to the feed element pattern have been used [26].

The mutual resistance given by (18) is derived from an idealized feed pattern and neglects higher-order modes and cross polarization. It relates to the small signal which appears at the port of a feed when an adjacent feed is excited. This effect is quite negligible for practical feed-array elements and (16) is a good approximation to the power radiated by the feed array. However, the effect of the array environment on the element pattern is much stronger. This effect, which may be termed mutual scattering, sets up the cross-polar mode and higher-order modes at the radiating aperture and has a pronounced effect on the cross-polar performance.

Much work has been carried out on mutual coupling in phased-array antenna systems using the concept of the active element pattern and the unit cell approach. These techniques do not apply to contoured-beam antenna feed arrays which have very non-uniform amplitude and phase distributions. The number of elements is typically smaller than in a phased array and the element size is larger. No electronic scanning takes place and the blindness effects of scanning phased arrays are of no concern. The concept of the embedded element pattern is more useful. The embedded feed pattern is defined as the pattern the feed radiates in the array environment with the feed element excited and all other elements terminated with their actual loads. Mismatches at the feed ports into the BFN have been noted to have a significant effect on the cross polarization - in particular for circular polarization.

A comprehensive study of mutual coupling in contoured-beam antenna feed arrays has been reported in [27]. The method of moments is used and the results apply to a finite number of circular waveguide feeds in a ground plane.

## 2.7. Spillover Loss Calculation

The spillover loss of a contoured beam will be lower than that of an element beam because of the array factor. The spillover loss is defined as the ratio of the feed power which hit the reflector to the total radiated feed power. The total power radiated by the feed array has been calculated (16). The element beams overlap and are usually not orthogonal. Therefore, the power intercepted and radiated by the reflector must be determined by integrating the total reflector far field. It is much simpler, however, to determine the power radiated by the reflector by integrating the Poynting vector across the reflector aperture  $A$  as proposed in [28]. We then obtain for the amplitude distribution in (4)

$$P_{rfl} = 4\pi \sum_{i=1}^N \sum_{j=1}^N a_i a_j^* [\alpha^2 A_1(k\alpha x_{ij}) + 2\alpha\beta A_{n+1}(k\alpha x_{ij}) + (n+1)^2 \beta^2 A_{2n+1}(k\alpha x_{ij})/(2n+1)], \quad (19)$$

where the pattern coefficients  $\alpha$  and  $\beta$  are defined above. The parameter  $x_{ij}$ , similar to (6), is the spacing between the beam centers in the uv-plane. The "cross correlation" pattern in (19) is wider than the element-beam pattern except in the case of uniform illumination. If the element-beam positions coincide with the nulls of the "cross-correlation" pattern, the beams decouple and become orthogonal. It appears that "losses" due to non-orthogonal beams are of a fundamental nature and may be related to different mechanisms in different antenna systems, e.g., BFN losses, spillover losses in reflector and lens systems and grating-lobe losses in arrays [29].

Figure 8 shows the spillover losses for a single element beam and a cluster of seven element beams. The feed spacing is kept constant equal to  $1.07\lambda$  and the beam spacing is increased by decreasing the focal length. The angle subtended by the reflector increases with increasing beam spacing, and the element beam spillover loss decreases. The dotted curve is the element beam spillover loss calculated accurately by a reflector antenna analysis program. The spillover loss of the 7-element cluster is much lower than that of the element beam and almost independent of the beam spacing. For a fixed reflector geometry, the beam spacing (or the beam crossover level) may be varied by changing the feed spacing. Table 3 gives the approximate spillover losses of both single and uniformly excited 7-element clusters of circular fundamental-mode and dual-mode feeds. The table gives the calculated spillover losses for  $\theta_p/\theta_0$  equal to 0.7, 1 and 1.3 corresponding to crossover levels of 1.5, 3, and 5 dB.



	Single feed			Cluster		
$\theta_B/\theta_3$	0.7	1.0	1.3	0.7	1.0	1.3
$TE_{11}$	5.7	3.1	-	1.5	1.3	-
$TE_{11}+TM_{11}$	6.8	4.1	2.3	2.6	2.4	2.1

Table 3. Spillover loss in dB.

A significant fraction of the large element-beam spillover for small beam spacings may be recovered by the array factor. For the dual-mode feed cluster, where the grating lobe losses are larger due to the larger feed size, the effect of the array factor on the spillover loss becomes very small for large beam spacings. A hexagonal array lattice will reduce the grating-lobe losses. For each feed type, a smaller feed spacing will slightly reduce the cluster spillover loss. This table indicates that there is a need for improved feed elements which utilize the focal-plane area more efficiently but without high mutual coupling. Another and probably more promising remedy might be to use improved reflector systems.

Even though the spillover loss of a contoured beam can be considerably reduced by the array factor, it can not be neglected. In the sense that this loss represents radiated power, it is very harmful in a frequency re-use antenna system if it is intercepted by this antenna or by other satellite structures and re-radiated as cross polarization back into the coverage or as high sidelobes into adjacent coverages. Such antenna-farm effects are difficult to predict as they depend upon the wide-angle radiation from the feed array and require special analysis software.

## 2.8. Scan Characteristics of Single Reflectors

Figure 9 shows isogain contours for the on-axis and some scanned element beams for a 3.2 m offset paraboloidal reflector at 4 GHz with an  $f/D$  ratio of 1.3. As a beam is scanned away from boresight, a gain loss and a beam widening occur. These beam degradations depend for a given scan angle upon the  $D/\lambda$  ratio, the offset angle  $\theta_0$ , the  $f/D$  ratio, and the aperture illumination. For the single offset paraboloidal reflector, where the scan degradation is due to astigmatism [30], the scan loss in dB for the scan angle  $\theta_{sc}$  may be approximated by

$$L_{sc} = C(n, \alpha_0) (D/\lambda \sin \theta_0 \sin \theta_{sc})^2, \quad (20)$$

with  $C(n, \alpha) = 0.1116 [(n+1)\alpha + 6(1-\alpha)/(n+2)(n+3)] / (1+n\alpha)$ . The parameters  $n$  and  $\alpha_0$  are the exponent and the edge taper of the reflector aperture illumination (4). For scan losses larger than 5 dB, (20) predicts too large losses. The scan loss is not a loss in radiated power as the BFN loss and the spillover loss. It represents a loss in the resolution or the beam-contouring capability of the reflector due to the widening of the element beams for large scan angles. Some scan degradation may be compensated for by a more complex feed array.

The scan losses of the center-fed paraboloidal reflector are for the same aperture diameter,  $f/D$  ratio, scan angle and aperture illumination order of magnitude less than those of the offset-fed reflector. The dominant aberration is due to coma, which has a minor impact on scan loss and mainly degrades the sidelobe performance [31]. As the total scan loss is determined by a combination of coma, higher-order astigmatism and spherical aberration, no simple expression exists for the scan loss in the center-fed paraboloidal reflector.

## 2.9. Polarization Considerations

Circular polarization was initially chosen for use in the international communications satellite system because of the Faraday rotation. When a linearly polarized signal traverses the ionosphere, the polarization plane undergoes a rotation. At 4 and 6 GHz, the maximum Faraday rotation is approximately 9 and 4° with opposite directions for transmit and receive (CCIR Report 551-1, Sect. 2.3.1). This choice of polarization was fortunate from the point-of-view of antenna technology. Circularly polarized offset reflectors do not generate cross polarization but exhibit a slight beam squint in the plane perpendicular to the plane of symmetry [32, 33]. The magnitude of this beam squint is given by the approximate formula

$$\Delta\theta = \text{Arcsin}(\lambda \sin \theta_0 / 4\pi f). \quad (21)$$

The direction of the beam movement depends upon the hand of the polarization. The Faraday rotation is inversely proportional to the square of the frequency and presents no problem above 10 GHz. Then, depolarization caused by rain becomes important and from the point-of-view of cross polarization, circular polarization becomes the worst possible choice. The shape of rain drops is generally spherical. However, the shape of falling rain drops becomes slightly elliptical due to the air resistance. The rain-induced attenuation and phase shift are maximum (minimum) for the polarization aligned with the major (minor) axis of the rain drop. No depolarization takes place for linear polarization aligned with the major or the minor axis of the rain drop. In practice, the depolarization is minimized for linear polarization aligned with local earth station vertical and horizontal (CCIR Report 564-2, Sect. 8.2). Thus for frequency re-use antenna systems in the 14/11 GHz bands and possibly even the 30/20 GHz bands, linear polarization should be used aligned with the average local vertical and horizontal within each coverage area [34]. However, this alignment will reduce the cross-polar isolation between adjacent coverage areas under clear-sky conditions as the polarizations then no longer are orthogonal.

In the single offset paraboloidal reflector, the feed axis is in general tilted to bisect the angle subtended at the focal point by the reflector in its plane of symmetry. This feed axis tilt generates two cross-polar lobes in a linearly polarized system [32, 35, 36]. The peak of the cross-polar lobes occurs in the plane perpendicular to the plane of symmetry. For a uniformly illuminated aperture, the peak cross-polar level relatively the peak copolar level is

$$E_{\text{cross}}/E_{\text{co}} = 0.36 \theta^* \tan \theta_0 / 2, \quad (22)$$

where  $\theta^*$  (in radians) is the semi angle subtended by the reflector. A tapered aperture illumination will decrease the cross polarization slightly. Any significant reduction requires either a more complicated feed element, a more complex feed array design, or that a gridded reflector be used.

### 3 OPTIMIZATION OF CONTOURED-BEAM REFLECTOR ANTENNA SYSTEMS WITH FEED ARRAYS

The synthesis of a contoured-beam antenna system may be divided into steps. Initially the fundamental antenna performance requirements such as

- coverage area(s),
- minimum coverage-area gain,
- maximum coverage-area gain slope,
- sidelobe and cross-polar isolation and
- frequency band

must be specified. Next, the antenna designer may identify the range of a number of antenna parameters such as

- reflector aperture size, and
- feed-array complexity

and examine the performance trade-offs versus the antenna system size and complexity. These trade-off studies require that optimum feed excitations be determined and the resulting contoured beams be analyzed for several antenna configurations. If these studies are carried out with simple analytic element-beam models, large savings can be realized in both human effort and computer time.

When a viable solution has been identified by these initial optimizations, an element-beam layout with half-power element beam beamwidth  $\Delta\theta_3$  and the element beam spacing  $\theta_3$  will be known. Given this element-beam grid, the feed-element size and the orientation of antenna on the satellite, an initial reflector antenna and feed array layout may be determined ensuring that no blockage occurs. Then, the detailed optimizations are carried out using element-beam data determined by an accurate reflector antenna analysis program. It is desirable to include as many potential error sources as possible. Degrading effects not predicted by the software can be included by means of measured data, e.g., of the patterns of the feeds embedded in the array. In case the results of the optimizations indicate that the performance requirements can not be met, these requirements or the range of antenna parameters being considered must be revised.

#### 3.1 Optimization Procedures

Many different, more or less rigorously based optimization procedures have been proposed to determine the feed excitations which provide the "best" contoured beam. It appears that no optimization procedure is complete. Usually the antenna designer specifies the reflector and the feed array geometry. Only then an optimization procedure determines the "best" feed excitations by optimization of the antenna performance, e.g., gain and isolation, over a finite number of pattern sample points. These sample points will be termed synthesis stations as they may not correspond to actual earth stations. A more complete optimization can be carried out by repeating the feed excitation optimization for a large number of antenna geometries. A general optimization procedure may optimize both element-beam grid and excitations but the usefulness of the results will be limited by the accuracy of the element-beam model. For practical antenna systems, it is still prohibitive and probably not desirable to include in the closed optimization loop a complete electromagnetic analysis with a.o. SPM tolerance analysis and feed-array mutual coupling analysis.

A contoured-beam synthesis is a power-pattern synthesis problem as opposed to a field-pattern synthesis problem. Furthermore, the power pattern is only specified in certain regions. The power pattern in the complementary regions of the far-field sphere and the phase pattern should be allowed to float in the optimization and take on any values which improve the power pattern in the regions of interest. Thus, even the apparently simple problem of determining the best feed excitations for a specified antenna geometry is a complex nonlinear problem and it can not be determined if a solution is a local or a global optimum. It appears often, however, that the optimum is quite flat so that small changes in antenna geometry, initial feed excitations input to the optimization procedure, stations locations, etc. may result in quite different feed excitations but very often only in small changes of the antenna performance. It is recommended to input different initial solutions to iterative optimization algorithms and to carry out sensitivity studies of the final solution with respect to excitation errors. The optimization is in general carried out on the copolar field only. In systems which implement frequency re-use by means of orthogonal polarizations, the required cross-polar performance is typically implemented by antenna designs which have low inherent cross polarization.

The following four sections describe a least-squares optimization procedure with a power constraint, the formulation of the minmax synthesis problem, a minmax optimization procedure which utilizes a general-purpose algorithm which recently has been extended to work more efficiently on contoured-beam synthesis problems, and recent progress with minmax or maxmin algorithms which have been developed specifically for contoured-beam reflector antenna synthesis. Many different approaches are described in the literature [e.g., 37-45].

#### 3.2 Least-squares Synthesis

The method, also known as the regularization method, is explained below using a matrix notation [46]. We define the matrix  $e = [e_{ij}]_{M \times N}$  where  $e_{ij}$  is the contribution of element beam  $j$ ,  $j = 1, 2, \dots, N$ , towards synthesis station  $i$ ,  $i = 1, 2, \dots, M$ . Two column vectors  $s = [s_i]$  and  $g = [g_i]$  contain the  $M$  excitations and the desired field on the  $M$  synthesis stations. The number of synthesis stations exceeds in general the

number of element beams and a solution can only be found in a least-squares sense. In order to optimize the minimum coverage area gain, the power input to the antenna must be constrained. Therefore we add the norm of the excitation vector  $e$ , which gives the incident power (16), to the least-squares pattern error by a Lagrange multiplier  $\alpha$ . The expression to be minimized is then

$$J = (e^T e^T - g^T)W(ea - g) + \alpha e^T e, \quad (22)$$

where the weight matrix  $W$  is a diagonal matrix and the superscript "T" denotes conjugate transposition. This gives the following matrix equation for the unknown excitations  $e$

$$(e^T W e + \alpha) e = a^T W g. \quad (23)$$

Initially, we only specify the amplitude of the desired field on the synthesis station by

$$g_1 = (G_0 p_1 f_{01})^{1/2}, \quad (24)$$

where  $G_0$  is a gain normalization constant,  $p_1$  the desired relative gain level, and  $f_{01}$  a path length correction factor if flux density rather than gain shall be optimized. As gain normalization constant we can use the peak achievable minimum coverage area gain (1) with a back off. This requires that the angular area  $\Omega$  of the coverage area be determined.

The relationship between the Lagrange multiplier  $\alpha$  and the incident power is established by means of the eigenvalue equation corresponding to (23). The matrix  $e^T W e$  is Hermitian and the positive real eigenvalues and the eigenvectors may be determined by standard techniques. We expand the unknown excitation vector in the complete basis formed by the eigenvectors. The expansion coefficients are determined in terms of the Lagrange multiplier  $\alpha$  which in turn is derived from the power constraint (16).

The specification of the desired field (24) implied a constant pattern phase. To remove this limitation in a heuristic way we include a phase factor  $\exp(j\phi_1^k)$  which is updated iteratively. At the  $k$ th step we determine the pattern phase from the excitations derived in the  $k-1$ th step by

$$\exp(j\phi_1^k) = \frac{\sum_{j=1}^N e_j^{k-1} a_{1j}}{\left| \sum_{j=1}^N e_j^{k-1} a_{1j} \right|}. \quad (25)$$

A new right-hand side of (23) is calculated and a new value of  $\alpha$  must be determined for each step in the phase iteration. However, the eigenvalues and the eigenvectors remain the same throughout the iteration. The iteration is terminated when the relative change of the least-squares error (22) decreases to a specified value.

### 3.3 Formulation of Minmax Synthesis Problem

The least-squares solution discussed above will in many cases be very good except, possibly, on a few critical synthesis stations. A minmax method may improve the performance on these critical stations generally at the expense of the average performance over all stations. The minmax optimization problem consists of finding the feed excitations  $e_j$  so that the realized power gain

$$p_{1i} = \left| \sum_{j=1}^N a_{1j} e_{spj} \right|^2 \quad (26)$$

minimizes the maximum value of the residual or pattern error

$$f_i = w_i |p_{1i} / (f_{0i} G_0) - p_i| \quad (27)$$

over all pattern constraints  $i$ , i.e., synthesis stations  $s$  and polarizations components  $p$ . In (26) and (27), the following notation has been used

$e_{spj}$	field towards station $s$ in polarization $p$ from element beam $j$ ,
$f_{0i}$	path length compensation factor towards station $s$ if flux density is optimized,
$p$	polarization selector equal to 1 or 2,
$p_i$	desired relative power level for pattern constraint $i$ . Often $p_i = 1$ for coverage-area constraints, $p_i = 0$ for isolation constraints and $p_i$ equal to a specified gain roll-off function with a back off if a reference pattern shall be enforced,
$s$	synthesis station number, $s = 1, 2, \dots, M_s$ , and
$w_i$	weight factor used to equalize coverage and isolation constraints.

The residuals for coverage area pattern constraints which exceed the gain goal are set equal to zero.

The minmax optimization problem consists of determining the feed excitation vector  $e$  which minimizes the maximum residual

$$f_{\min\max} = \max_i f_i. \quad (28)$$

The corresponding least-squares error (22) minimizes the average deviation over all stations. The weight factors  $w_i$  and  $M_i$  are used to increase the magnitude of the residuals in isolation areas to correspond to the magnitude of the copolar residuals in the service area. If the minimum coverage area gain is MCAG, the maximum sidelobe (cross-polar) level is MSL and the coverage area weight factor is unity, the minmax sidelobe (cross-polar) weight factor

$$w = (G_0 - MCAG) / MSL \quad (29)$$

will equalize the residuals. The weight factors behave slightly differently in the minmax and in the least-squares optimization. The best agreement between the two optimizations is obtained if  $w_i = w_i^{1/2}$ .

### 3.4 Synthesis by General Minmax Algorithms

The general minmax algorithm is an extension of the iterative method described in [47]. Let the column vector  $x_k$  represent the excitation vector at the  $k$ th step of the iteration,  $f_i(x_k)$  the associated error or residual (27) at the  $i$ th pattern constraint, the column vector  $b_{ik}$  an approximation to the gradient of the residual with respect to the excitations and the column vector  $h$  an increment to the excitation vector. Then,

$$F_k(h) = \max_i (f_i(x_k) + b_{ik}^T h) \quad (30)$$

is a linear approximation of the minmax pattern error (28) for small values of  $h$   $h = \max |h_j|$ . At each step of the iteration, the linear subproblem (30) is solved by linear programming subject to a bound on the solution,  $h \leq \lambda_k$ , giving the solution  $h_k$ . The vector  $x_k + h_k$  is accepted as the new approximate solution if the non-linear objective function has decreased. Otherwise the step is rejected and repeated with a smaller  $\lambda_k$ . The bound  $\lambda_k$  reflects the region near  $x_k$  where (30) is a reasonable approximation to the original non-linear problem and it is adjusted automatically during the iteration. The approximations to the gradients are updated using a rank-one formula by Broyden [48],

$$b_{ik+1} = b_{ik} + (f_i(x_k + h_k) - b_{ik}^T h_k) / (h_k^T h_k) h_k. \quad (31)$$

In a contoured-beam optimization, the number of synthesis stations can be large in particular when a global envelope constraint is enforced as in Section 4.2 or when several contoured beams with common excitations are optimized simultaneously as in Section 4.3. A significant amount of storage is needed for the derivative matrix and large linear programming problems must be solved at each step of the iteration. In practical problems, however, the number of stations where the pattern error (residual) attains the maximum value is small compared to the total number of stations. Therefore at each step, we identify the stations within a specified range of the largest residual. These stations define the overall pattern error  $F$  in a neighborhood about  $x_k$  and are called the active stations. Working mainly only with the stations within a range of the largest residual, we realize large savings in storage and computing time whereas the convergence properties remain the same. The derivative matrix is stored and updated only for the active stations - typically about 1/5 and less of all stations. The size of the linear programming subproblems are reduced by the same proportion. However, all stations are still checked at each step of the iteration and the active set is updated as required. The gradients of the residuals which are active at both  $x_k$  and  $x_k + h_k$  are updated by (31). For the newcomers we use a difference approximation since we have no estimate  $b_{ik}$  of the gradient at  $x_k$ .

In case of a singular problem, the iteration may become slow because the process is caught in a long valley with steep sides [49]. It is a characteristic of a singular problem that with  $N$  excitation variables, the number of worst stations where the residuals are equal to the maximum residual will be less than  $N+1$ . The number of worst stations is generally considerably less than the number of active stations used in the iteration and less than the number of excitation variables even in the final stages of the iteration. Thus, most contoured-beam synthesis problems appear to be singular. When we decide that the process has been caught, we apply a "special" non-descent iteration to bring the process out of the valley. It can be proven that the extended method theoretically has the same convergence properties as the processor whereas in practice the new method is considerably faster and can handle much bigger problems within the same computer memory.

### 3.5 Specialized Minmax Algorithms

The general minmax algorithm provides the maximum degree of freedom in the design. We may choose to optimize only the amplitudes, only the phases or both the amplitudes and the phases of the excitations with only small changes of the software. Also, the location of the element beams may be optimized or degrading effects such as BFN frequency dispersion and BFN random errors could be accounted for. However, the general algorithm does not take advantage of the special properties of a specific contoured-beam synthesis problem as opposed to, e.g., microwave network synthesis problems. The general algorithm does not use the closed-form analytic derivative of residuals which are available in special cases but uses approximations derived by finite differences and Broyden's formula. These approximations are both more time consuming and more inaccurate. They may result in a reduced rate of convergence in the final stages of the iteration, where a large linear programming problem may have to be solved at each step.

A simple minmax algorithm has been proposed by us in [50] to be applied in cases with amplitude only or both amplitude and phase excitation optimization. For the excitation normalization (16) with no loss, the derivative of the  $i$ th residual  $f_i$  (27) with respect to  $a_j$  is given by

$$f_{ij}' = 2 \left[ \left( \sum_{j=1}^N a_{ij} e_j \right) a_{ij}^* - \left| \sum_{j=1}^N a_{ij} e_j \right|^2 e_j \right]. \quad (32)$$

The factor  $w_i / (f_{0i} G_0)$  has been suppressed. It is further shown in [50] that the gradients  $\nabla f_i$  are perpendicular to the excitation vector. At each step in the iteration, the active stations are identified and a best search direction is found by solution of a system of linear equations. The length of the incremental vector  $\Delta a_k$  to be added to the excitation vector  $a_k$  in the  $k$ th step is determined very efficiently by a linear search.

We found that each step in the iteration proceeds extremely fast compared to the general minmax algorithm and that very often good results are obtained. The range of the residuals which define the active stations gradually decreases during the iteration. In our implementation, the method sometimes requires very many iterations to terminate. When the number of active stations becomes equal to the number of excitations, the linear system of equations used to determine the search direction becomes singular and the iteration is forced to terminate.

Recently, the convergence of the method has been considerably improved by determining the best search direction by solving a linear programming problem and improving the linear search [51]. Only the worst stations are considered in the search direction determination. The results indicate that order-of-magnitude saving in computer time may be realized when closed-form derivatives are used instead of

derivative approximations. A possible future extension may be to implement a hybrid algorithm which would use analytic derivatives whenever they are available and derivative approximations otherwise.

#### 4 SYNTHESIS EXAMPLES

The optimization procedures described in Section 3 have been implemented in a synthesis software. This section gives three examples of results obtained with this software package. Other recent design examples include dual-mode antenna optimization [52] and an offset side-fed Cassegrain antenna with ten contoured beams [53].

##### 4.1 Antenna without Stringent Sidelobe Specifications

###### 4.1.1 Contoured-beam antenna specifications

The optimization of a contoured-beam reflector antenna system for a European Communications Satellite coverage is provided to illustrate the design procedure. The antenna is a transmit antenna with the requirements outlined below:

Frequency band:	10.7-11.7 GHz
Polarisation:	RHCP
Min. coverage area gain including	
RFN loss and spillover:	28 dBi
Max. pattern slope in coverage area:	5 dB/degree
Max. copolar sidelobe level:	-20 dB
Min. cross-polar isolation in coverage area:	30 dB

An area coverage including a  $0.2^\circ$  pointing error was specified. The number of feeds was anticipated to be between 20 and 30 and the antenna envelope should be compatible with the ECS platform and the Ariane launcher. The feed element was chosen to be a small conical horn placed in a hexagonal grid with an element spacing of  $1.07\lambda$  at the center frequency. The coverage was composed of four isolated stations and a large area specified by a piece-wise linear contour. The area specification was converted into discrete station specifications by adding synthesis stations inside the piece-wise linear contour. It was found that a spacing between the samples in the uv-plane of

$$\delta_{uv} = 0.01 \Delta\theta_j,$$

(33)

with the element beam beamwidth  $\Delta\theta_j$  in degrees, gave a reasonable trade off between accuracy and computation time. This corresponds to about 60 per cent of the maximum Nyquist sampling spacing  $\lambda/D$  and allows for truncation effects near the edge of the coverage, the tilt of the effective aperture plane along the plane of the reflector rim curve, etc. When a large range of element beam beamwidths is being considered, it becomes necessary to use more than one coverage sampling spacing. Figure 10 shows the 58 synthesis stations representing the ECS coverage for the half-power beamwidths  $2^\circ$  and  $1.5^\circ$ . The spacing (33) between the internal samples is 0.015 corresponding to  $0.86^\circ$ . For the half-power beamwidth equal to  $1.2^\circ$ , the sample spacing was reduced to 0.012 or  $0.69^\circ$ , and 81 synthesis stations were obtained.

###### 4.1.2 Initial optimization with analytic element beams

The preliminary design trade-off was carried out with the analytic beam model for the beamwidths  $\Delta\theta_j = 2.0^\circ, 1.5^\circ$  and  $1.2^\circ$  to determine approximately the reflector size and the number of feed element. In each case, the beam spacings  $\theta_g = 1.2, 1.1$  and  $1.0 \Delta\theta_j$  were considered. The initial beam grids set up included all element beams with a distance less than  $\Delta\theta_g$  from the nearest stations. With  $\Delta\theta_g = 1.25, 1.00$  and  $0.80 \Delta\theta_j$ , a total of 27 initial beam grids were considered. In each case, the flux densities on the Earth were optimized taking into account the path length differences between the satellite and the synthesis stations. An initial set of real excitations was obtained by the least-squares algorithm and used as starting point for the minmax algorithm which also varied the antenna pointing and the beam lattice orientation and spacing. All the excitations are real-valued because the element beams are real-valued.

The configuration with  $\Delta\theta_j = 1.5^\circ, \Delta\theta_g = 1.65^\circ$  and initial value  $\theta_g = 1.65^\circ$  was found to give the best compromise between antenna complexity and performance. However, the optimum is quite flat. Figure 11 shows optimized the element-beam lay-out, which includes 24 element beams, and the isogain contours of the optimized contoured beam 1, 2, 3, 5, 10, 15, 20, 25, and 30 dB below peak gain. The number of "active" stations in the minmax optimization is 17, which is well below 24, the number of element beams. This indicates that we are dealing with a singular problem. Only four least-squares residuals exceeded the minmax residual. The associated synthesis stations belonged to the set of "active" stations in the minmax solution. The analytic beam model provides an initial antenna geometry shown in Figure 12 as seen from the dish. The feed lay-out is the image of the element-beam lay-out in Figure 11.

###### 4.1.3 Final optimization with PO/GTD element beams

The initial antenna geometry was further optimized using accurate element beam data determined by PO, GTD and Whittaker reconstruction. These beams are complex-valued so that both feed excitation amplitudes and phases were optimized. The initial least-squares feed excitations were determined using iterative updating of the pattern phase and used as starting point for the minmax excitation optimization. The optimized excitations were inspected for weakly excited feeds. These feed were eliminated and the excitations of the remaining feeds re-optimized. Table 4 summarizes the principal results with an assumed RFN loss of 1.4 dB.

	Least-squares	Minmax		
	Min flux	Min flux	Min gain	Spillover
Analytic beam model:				
all 24 beams	27.85	28.31	28.09	-1.02
PO/GTD beams from GRASP:				
all 24 beams	28.21	28.40	28.17	-1.21
-beams 18, 20 and 22	27.81	27.97	27.73	
-beams 20 and 22	28.15	28.26	28.02	

Table 4. Results of ECS optimizations. Levels in dB and dBi.

Several observations may be made. Firstly, the least-squares solution is in all cases quite good compared to the much more time consuming minmax solution. Secondly, the prediction obtained by analytic element-beam model is very close to the one obtained using a much more accurate element-beam model. The element-beam model is quite sensitive to the value assumed for the feed quality number  $q_{8,10}$ . Thirdly, deleting feed or element beam 18 has a significant impact on the solution so that caution should be exercised when feeds are eliminated. The optimization should begin with a generous number of feeds which gradually are removed. Each time, the excitations of the remaining feeds must be re-optimized. Figure 13 and 14 show the element beam half-power contours and the contourad-beam leakage curves derived by means of PO, GTD and Whittaker reconstruction at 10.7 and 11.7 GHz. The decrease of element-beam half-power beamwidth with increasing frequency may be noted. The agreement with the analytic beam model in Figures 11 is good except for a slight widening of the low level contours in Figure 13 and 14 due to the scan aberrations neglected in the analytic element beam model.

The finite tolerance of the BFN, mismatches, frequency dispersion in the BFN and mutual coupling will cause the realized feed excitations to deviate from the optimum values determined by the synthesis software. Excitation errors not accounted for will degrade the antenna performance. Some of the degrading effects may be included in the pattern optimization. Thus, it has become common practice to carry out the excitation optimization simultaneously at both edges of the frequency band if the antenna is required to operate over any significant bandwidth. Table 5 summarizes the results of computer simulations of the effect of random excitation errors on the minimum coverage area gain. For the ranges of errors and the configuration considered, amplitude errors seem to be more serious than phase errors. For practical applications, the average excitation error should be less than a few tenths of a dB in amplitude and  $5^\circ$  in phase.

Peak excitation		Gain degradation due to		
amplitude	phase	amplitude	phase	amplitude
error	error	only	only	and phase
0.3 dB	$3^\circ$	0.19 dB	0.00 dB	0.17 dB
0.6 dB	$6^\circ$	0.32 dB	0.09 dB	0.44 dB
0.9 dB	$9^\circ$	0.61 dB	0.16 dB	0.66 dB
1.2 dB	$12^\circ$	1.16 dB	0.24 dB	0.88 dB
1.5 dB	$15^\circ$	1.12 dB	0.32 dB	-

Table 5. Degradation of the minimum coverage area gain as a function of random excitation amplitude and phase errors.

#### 4.1.4. Measurement of the minimum coverage area gain as a function of random excitation amplitude and phase errors.

In the European Space Agency's COBRA (contoured-beam reflector antenna) program, the prototype of the antenna described above was designed, manufactured, integrated and tested [54]. The rms deviation of the measured amplitude and phase excitations from the nominal values are indicated in Table 6 at the edges and the center of the frequency band.

Frequency	Amplitude	Phase
10.7 GHz	0.58 dB	$5.6^\circ$
11.2 GHz	0.24 dB	$5.9^\circ$
11.7 GHz	0.59 dB	$6.5^\circ$

Table 6. RMS deviations of excitations at feed ports.

Figure 15 shows in full line the measured copolar contours 1, 3, 5, 10, 15 and 20 dB below peak gain at the edges and the center of the frequency band. The calculated contours 3 and 20 dB below peak gain are superimposed in dotted line. The agreement is fair apart from 11.7 GHz where strong mutual coupling effects between the small circularly polarized feeds not accounted for in the analysis are believed to degrade the minimum coverage area gain from 28.1 to 25.6 dBi at Barcelona. The cross-polar discrimination was better than 28 dB at the design frequency [55].

#### 4.2 Impact of Global Sidelobe Constraint on Frequency Re-use Antenna System

The interference between the different satellite communications systems has in the past been controlled by coordination such that any new system would not obstruct the existing systems. In the future, it is likely that communications satellite antennas must meet reference pattern specifications similar to those which already apply to earth-station antennas and direct-broadcast satellite antennas. This will conserve the finite available frequency bandwidth and geostationary arc. Such antenna reference patterns are difficult to define for a contoured beam, which may be of a complicated shape. They will complicate the antenna design process as many more synthesis stations must be considered. This example investigates the potential impact of a global pattern envelope constraint on an antenna similar to the Intelsat VI 4-GHz hemi-cone antenna already considered in Section 2.2. This antenna already meets stringent sidelobe requirements in the adjacent frequency re-use coverage areas.

#### 4.2.1. Sidelobe envelopes of current antennas

Figure 16 shows the calculated isogain contours at MCAG (the minimum coverage area gain) and 27, 30 and 33 dB below MCAG for the Intelsat VI Indian Ocean region (IOR) zone 2 beam at the lower edge of the 4-GHz band where the problems with sidelobes and the roll-off of the pattern from the coverage area are the most severe. The four spatially isolated zone beam coverages are indicated by the four piece-wise linear composite coverages. We note that fairly high sidelobes extend from zone 2 towards zone 4. The figure also shows a number of linear segments which extend from about  $1^\circ$  inside the zone beam coverage to about  $7^\circ$  outside. Figure 17 shows the superimposed cuts along these gain roll-off pattern traces. The horizontal angular scale gives the distance from the point where a gain roll-off pattern trace intersects the MCAG contour. No traces away from the Earth are considered. In these directions, the sidelobes are much higher. The full (dotted) line near  $0^\circ$  represents the maximum (minimum) envelope of the pattern traces. The full (dotted) line further away indicates the envelope of most (all) of the traces. The dot/dash line indicates the additional degradation caused by the calculated line length dispersion in the beam-forming network. In a practical antenna, several other imperfections will degrade the performance.

#### 4.2.2. Gain roll-off optimization

Figure 18 indicates the location of the synthesis stations used in the optimization. Initially, no gain roll-off stations (indicated by 7) were used. The optimization then uses 176 stations and 36 beams. It is carried out at both band edges simultaneously so that the number of pattern constraints is twice the number of stations. Figure 19 shows the resulting gain contours at the lower edge of the band with isogain contours at the MCAG level and 27, 30 and 33 dB further down. The levels do not include losses and give directivity. The minmax solution is superior to the least-squares solution on the worst stations and improves in this case the MCAG level by 1.2 dB. The sidelobes are higher than in Figure 16 but the fit to the desired composite coverages is tighter.

Figure 20 shows the optimized gain contours obtained for carefully selected sets of gain roll-off stations. Initially, gain roll-off stations were set up from the edge of the composite coverage with a maximum tolerable field level calculated from a gain roll-off reference curve. This may lead to the specification of critical synthesis stations which with the minmax algorithm would destroy the overall antenna performance. With the gain roll-off curve used in the optimization, it was found that no gain roll-off station should be placed closer to the coverage area than  $3^\circ$  for this zone beam. Also, no stations should be placed in (the polar) regions where no feed is available for controlling the pattern. The resulting 265 synthesis stations are shown in Figure 18 with deleted gain roll-off stations marked by circles. The sidelobe performance has been considerably improved while the MCAG level has been reduced by only a few tenths of a dB. However, the gain slope of the edge of coverage has been degraded. Increasing the number of feeds slightly will improve the MCAG level and the gain slope but seem to have little impact on the sidelobe performance.

#### 4.3. Reconfigurable Antennas with Shared Excitations

Increasing satellite lifetimes of 10 to 14 years have accelerated the need for the same spacecraft to be able to provide different services over different coverages at different times. The full range of possible future requirements to a spacecraft cannot be anticipated for such a long time and can probably only be met by a fully reconfigurable antenna system. Such antenna systems do not require any special synthesis software as the antenna may be optimized separately for each coverage requirement. However, they are excessively complex and expensive to implement, and in practice, less complex and more reliable systems with limited reconfigurability are implemented, e.g., as in the Intelsat C-band hemi/zonal antenna systems by means of switches. Such a case is considered in this section. A set of shared excitations is optimized to meet the Intelsat VI Atlantic and Indian Ocean region zone 2 beam coverage requirements. The shared excitations would then be combined with separate sets of unique excitations for each ocean region. In each ocean region, the sidelobe requirements in the adjacent zone beam coverages are included.

Figures 21 and 22 show the 185 synthesis stations used for the Indian Ocean region zone 2 beam optimization and the 218 synthesis stations used for the Atlantic Ocean region zone 2 beam optimization. The minimum coverage area gain is optimized subject to meeting a sidelobe isolation of more than 30 dB in the adjacent zone beam coverages. First, the two zone beams will be optimized independently of each other. Then, the two zone beams will be optimized simultaneously sharing any element beams with identical excitations.

##### 4.3.1. Optimization with separate excitations

A initial optimization is carried out for each coverage separately. Figure 23 and 24 show the contour plots of the two optimized zone beams. The optimizations are carried out using the analytic element-beam model. Isogain contours are shown through the minimum coverage area gain level and 20, 25, 30 and 35 dB further down. The minmax pattern error is slightly larger for Indian Ocean region zone beam.

##### 4.3.2. Optimization of shared excitations

At this stage, the two sets of excitations obtained by the optimization of each ocean region separately are inspected and each excitation is assigned to one of the following three BFNs:

BFN 1 generates the excitations only used by the Indian Ocean region zone beam,

BFN 2 which generates the excitations only used by the Atlantic Ocean region zone beam, and

BFN 3 which generates the excitations which are shared for the two zone beams.

The total number of element beams is 46. The breakdown of the element beams and the excitations between the two zone beams and the three BFNs is given in Figure 25. The combined synthesis problem consists of 61 different excitations and 403 synthesis stations. In this case, the synthesis is carried out so that the power division between BFN 1 and BFN 3 for the Indian Ocean region zone beam and between BFN 2 and BFN 3

for the Atlantic Ocean region zone beam are optimized by the program. Figure 26 and 27 show the contour plots of the two optimized zone beams with shared excitations for the analytic element-beam model. The minimum coverage area gain (or rather directivity) and the minimum sidelobe isolation are listed in Table 7 for the case with separate excitations and with shared excitations. The table gives data obtained both with the analytic element beams and with element beams calculated by an accurate reflector analysis program.

	Analytic element beams		PO/GTD element beams	
	Separate	Shared	Separate	Shared
Min. gain:				
Indian 2	26.18 dBi	26.19 dBi	25.91 dBi	26.00 dBi
Atlantic 2	25.58 dBi	25.36 dBi	25.32 dBi	25.17 dBi
Min. isolation				
Indian 2	33.09 dB	33.10 dB	32.29 dB	32.55 dB
Atlantic 2	33.81 dB	33.10 dB	33.09 dB	32.55 dB

Table 7. Performance with separate and with shared excitations.

Thus, no degradation has taken place for the Indian Ocean region zone beam which had the largest minmax residual. The performance of the Atlantic Ocean region zone beam has been "equalized" so that minmax residuals now are identical for the two zone beams. The agreement between the results obtained by analytic beams and PO/GTD beams is surprisingly good.

## 5 SHAPED CONTOURED-BEAM REFLECTOR ANTENNAS

In this Section we consider an alternative contoured-beam reflector antenna requiring only a single feed. The surface of the offset reflector is shaped so that the modified wavefront along the original reflector surface provides the desired wavefront. The deviation from the paraboloidal surface is so small that the amplitude distribution along the original reflector surface remains essentially undistorted and only negligible cross polarisation is generated. The similar surface shaping technique has previously been utilized for a Japanese experimental Ke-band communications satellite [56, 57].

### 5.1 Optimisation of Aperture Phase Distribution

In our version of the synthesis technique, the phase of the aperture field is expanded into Zernike or circle polynomials, i.e.,

$$\Phi(\rho, \phi) = \sum_{n=1}^N \sum_{m=-n}^n c_{mn} a_{mn}^{jm\phi} R_{mn}(\rho), \quad (34)$$

where  $\rho$  and  $\phi$  are the polar aperture coordinates and  $c_{mn} = c_{mn}^*$ . The Zernike or circle polynomials find use in optics for orthogonal expansions in circular apertures. The polynomials are simply related to the aberrations such as spherical aberrations, astigmatism, coma, etc. [59]. Rapid calculation of circle polynomials is possible by recursion. The expansion coefficients  $c_{mn}$  take the role of the feed excitations in the array-fed reflector. For a particular set of expansion coefficients, the field is calculated on all stations by a simplified and fast physical-optics integration across the deformed reflector surface. The general minmax routine described in [47] is used to determine an set of expansion coefficients which optimise the gain on all synthesis stations. The procedure has also been utilised to synthesise elliptical beams with very low sidelobes [58].

When an optimum phase distribution has been determined, the shaped reflector is derived by a ray-tracing procedure from the offset paraboloid used as initial solution. The optimisation of the phase expansion coefficients is numerically more difficult than feed excitation optimisation. The element beams are almost orthogonal and varying one feed excitation affects only few stations. The phase expansion coefficients, on the other hand, interact nonlinearly and varying one coefficient will affect all stations but by very small amounts.

The reflector shaping procedure has been applied to the ECS coverage already considered in Section 4.1. The design was carried out at 11.2 GHz using the offset paraboloidal reflector similar to the one considered in Section 4.1 as starting point. An optimized surface deformation which generates the contoured beam is indicated by the three-dimensional plot in Figure 28. The optimized shaped reflector antenna system was analysed by physical optics. Figure 29 shows the calculated copolar isogain contours at the edges of the frequency band 1, 2, 3, 5, 10, 19, 20, 21 and 30 dB below peak gain. The calculated minimum coverage area directivity of 28.81, 29.21 and 28.84 dBi at 10.7, 11.2 and 11.7 GHz compare favorably with the corresponding minimum coverage area gain values of 27.93, 28.18 and 28.10 dBi of the array-fed reflector. No attempt was made to suppress the sidelobes. The antenna was assumed to be circularly polarised. The cross-polar performance was found to be very sensitive to the cross-polar performance of the feed. Even low levels of feed cross polarisation would generate "hot spots" of cross polarisation in the reflector far field.

### 5.2 Measurements on Shaped Contoured-beam Reflector

The shaped reflector contoured-beam reflector was re-optimized with a smaller offset angle  $\theta_0$  to allow the use of a linearly polarised feed horn which was available. A modified aperture phase expansion was determined with significantly lower sidelobes at the cost of a reduced minimum coverage area gain. The shaped reflector was manufactured for the scaled frequency band 16.4 - 18 GHz in order to reduce the reflector size to be within the limitations set by the surface machining equipment available. The antenna measurements were carried out at the spherical near-field test range at the Technical University of Denmark. During the antenna measurements, the feed horn was found to move and it was necessary to



strengthen the feed support structure. Figure 30 compares over an extended angular range the measured and the calculated co- and cross-polar pattern cuts along the planes of maximum and of minimum beamwidth. The agreement is excellent apart from the angular region  $7^\circ < \theta < 14^\circ$  in the plane of the narrow beamwidth, where scattering from the enlarged feed support structure appears. The measured minimum coverage area directivity of 27.3 dBi occurs at the high end of the band where the predicted value is 28.1 dB. The shaped reflector antenna holds significant advantages over the array-fed reflector antenna for applications where no reconfigurability and only a single beam is required. No BFN is required and complicated mutual coupling effects in the feed array are avoided. The BFN and the spillover losses are absent or drastically reduced so that the gain delivered can be higher. Shaped reflector antenna systems are in rapid development and major future progress is likely with the recent advent of rigorous methods [60]. Work is also being carried out on dual reflectors [61].

## 6 DUAL REFLECTOR SYSTEMS WITH SMALL SCAN DEGRADATIONS

The dual offset Cassegrain or Gregorian reflector systems permit cancellation of the cross polarization for a linearly polarized on-axis beam [62]. This is achieved if the axes of the feed, the subreflector and the main reflector are adjusted according to the condition

$$\tan \gamma/2 = 1/M \tan \phi/2 \quad (35)$$

where  $\gamma$  is the angle from the main reflector axis to the subreflector axis and  $\phi$  the angle from the subreflector axis to the feed axis (see Figure 31). The value of the subreflector "magnification"  $M$  determines the subreflector type:

- (1)  $M > 1$ : The subreflector is the convex branch of a hyperboloid with the eccentricity  $e = (M + 1)/(M - 1)$ . This is a conventional dual offset Cassegrain.
- (2)  $0 < M < 1$ : The subreflector is the concave branch of a hyperboloid with  $e = (1 + M)/(1 - M)$ . This is either the front-fed or the side-fed dual offset Cassegrain discussed below.
- (3)  $M < -1$ : The subreflector is an ellipsoid with  $e = (M + 1)/(M - 1)$ . This is a conventional dual offset Gregorian.

Dual offset reflector configurations which fulfil the condition (35) have no first-order astigmatism which otherwise is the dominant aberration in offset reflector antennas [30]. Conventional compensated dual offset Cassegrain and Gregorian antennas,  $M > 1$  and  $M < -1$  above, are difficult to design with no blockage for large scan angles, and the feed array is larger than in the case where the main reflector is used as a single reflector. These problems are reduced when  $0 < M < 1$  for the two different configurations in Figure 32, which both are designed for a  $\pm 10^\circ$  scan. In Figure 32a, the feed array is located in front of both the subreflector and the main reflector and the system has been termed the front-fed offset Cassegrain (FFOC) antenna system [63]. In the second configuration in Figure 32b the feed array and the subreflector are placed on either side of the main reflector. This system has been termed the side-fed offset Cassegrain (SFOC) antenna system [53]. Both the FFOC and the SFOC have unique scan properties over the full  $\pm 10^\circ$  field-of-view due to the large focal length of the main reflector. The large field-of-view requires that the subreflector size be comparable to the main reflector size. For contiguous Earth coverage, the feed array size is excessive and the systems may not be competitive to an array antenna. However, the inherent property of a reflector antenna system of associating single feeds with high-gain spot beams makes the configurations shown in Figure 32 very attractive candidates for meeting partial Earth coverage requirements with many high-gain beams.

### 6.1. Scan Properties of the FFOC and the SFOC

Figures 33 and 34 show the calculated principal co- and cross-polar pattern cuts for secondary beams radiated by small linearly polarized conical horns placed at the locations which are corresponds to beams on axis and with  $10^\circ$  downward scan,  $10^\circ$  upward scan and  $10^\circ$  lateral scan. The diameters of the feeds are  $1.8 \lambda$  (FFOC) and  $2.2 \lambda$  (SFOC) and correspond to a beam spacing of 92 per cent of the beam width. This is close to the spacing which gives the highest gain at the cross-over level for multi-beam applications where each beam is radiated by a single horn. In each case, the feed location is optimized to minimize the phase errors and the feed axis is aligned so that the central ray hits the center of the main reflector surface. For both the FFOC and the SFOC, the diameter  $D$  of the projected aperture is  $120 \lambda$ . The small feeds provide only a slight aperture taper. This situation exhibits both the highest sidelobe and the largest scan losses. The calculated peak directivity, peak sidelobe level and peak sidelobe level for the two configurations are given in Table 8. The peak directivity is broken up into a number of efficiencies

$$G_{sc} = \eta_{ap} \eta_{ar} \eta_{sp} G_0 \quad (36)$$

where

- $\eta_{ap}$  is the apillover efficiency, (the fraction of the feed power which hits the main reflector),
- $\eta_{ar}$  is the relative projected area (the ratio of the area obtained by projecting the main reflector rim into a plane perpendicular to the direction of the scanned beam and into a plane perpendicular to the direction of the on-axis beam),
- $\eta_{sp}$  is the aperture efficiency including loss due to phase errors, amplitude taper and cross polarization. (This loss is dominated by the phase errors associated with the scan as the aperture illumination is almost uniform), and
- $G_0$  is the peak achievable directivity  $4\pi/\lambda$  (51.53 dBi for  $D = 120 \lambda$ ).

Beam direction	$\eta_{ap}$ dB	$\eta_{ar}$ dB	$\eta_{ap}$ dB	Directivity dBi	Sidelobe dB	Cross-pol dB
FPOC:						
on axis	-4.62	0	-0.03	46.88	-28.38	-48.68
10° downward	-3.48	+0.75	-2.28	46.52	-32.63	-38.06
10° upward	-5.87	-1.07	-1.55	43.04	-23.14	-35.52
10° sideward	-4.78	-0.07	-0.63	46.05	-33.63	-37.40
SFOC:						
on axis	-5.09	0	-0.02	46.42	-28.01	-54.78
10° downward	-4.68	+0.34	-0.36	46.83	-33.28	-40.20
10° upward	-5.72	-0.52	-0.24	45.05	-29.88	-37.49
10° sideward	-5.29	-0.07	+0.03	46.20	-30.25	-41.56

Table 8. Peak directivity budget and peak sidelobe level of FPOC and SFOC.

Due to the relatively small feed size and small angle subtended by the subreflector rim, the feed spillover is large. The large scan causes a significant change of the relative projected area. As the main reflector is closer to vertical for the SFOC than for the FPOC, the area loss for upwards scan is smaller for the SFOC. The aperture efficiency is, except for the on-axis beam, dominated by phase errors, i.e., even aberrations. We see that the effect of these phase errors is smaller for the SFOC. The smaller even aberrations of the SFOC also follow from the pattern in Figures 33 and 34. The sidelobe and the cross polarization are lower for the SFOC than for the FPOC. As the aperture diameter increases, e.g., to  $240\lambda$ , the superiority of the SFOC with respect to electric performance is accentuated. For both configurations, the even losses are higher for beam scanning in the plane of symmetry than in the perpendicular plane. In the plane of symmetry, the scan losses due to phase errors are slightly higher for downward scan, but they are compensated by reduced spillover and area losses. In a practical design, the antenna axis should be repositioned to equalize the overall even loss in all directions or give preference to critical areas in the coverage. Comparisons with the single offset paraboloidal reflectors show that the FPOC achieves similar scan performance as a single offset parabola with an  $f/D$  ratio equal to about 2.6. The SFOC is comparable to a single offset parabola with an  $f/D$  of about 5.6.

More details including the initial design of a feed array to generate 10 contoured beams out of the SFOC are given in [53].

## 7 CONTOURED-BEAM ARRAY ANTENNAS

Table 2 indicated the significant growth of the feed array size for each new INTELSAT spacecraft series. If the trend towards larger and radiating apertures and  $f/D$  ratios continues, the feed array size will ultimately exceed the reflector size and the antenna mass and volume requirements will have a tremendous impact on spacecraft design and launch cost. Thus, it may become advantageous to "discard the reflector and turn the feed array towards the Earth" and use it as a directly radiating array.

### 7.1 Array Excitation Optimization with Pencil Element Beams

We consider the planar array configuration in Figure 35. It consists of  $N_s$  elements in a hexagonal lattice. We define  $N_b$  element beams radiated by the array in a hexagonal lattice over the field-of-view shown in Figure 36. The half-power beamwidth of the element beams is determined by the array diameter  $D$  and is for a uniform array illumination approximately  $\Delta\theta_2 = \lambda/D$ . The optimization of the array excitations to meet specified coverage and isolation requirements is carried out by the algorithms discussed in Section 3 but indirectly by optimizing the excitations of the element beams. Then, the array excitations are calculated from the element-beam excitations. This optimization via the element beams is much more efficient than a direct optimization of the array excitations because

- (1) the number of element beams is in general much less than the number of array elements, and
- (2) the element beams are almost orthogonal to each other over the far-field pattern in the sense their pattern only overlap little as opposed to the array element patterns, which overlap completely. As a result, the optimization is a more well behaved problem.

The number of array elements for a given aperture size is determined by the requirement that no grating lobes fall in the field-of-view. The antenna designer may choose the element-beam positions independently for each coverage area of a multi-beam array antenna. This is not possible for a reflector antenna. The aperture size in wavelengths and the angular extent of the coverage area determines the number of the degrees of freedom of the synthesis problem [64]. In most cases, the beam spacing can be chosen a little larger than the beam width as in the case of a reflector antenna. No advantages are gained by choosing the beam spacing significantly smaller than the beam width.

Figure 37 shows an optimized contoured beam for an array consisting of 169 conical horns with a diameter of  $2.84\lambda$ . The array diameter is identical to the size of the INTELSAT VI hemi-cone reflector. By gradual removal of weakly excited array elements followed by re-optimization of the remaining excitations, the number of array elements can be reduced significantly. A full account of the design procedure and many design cases have been given in [65].

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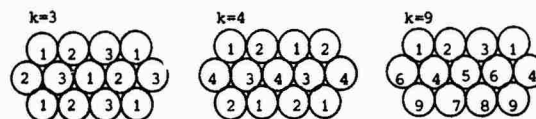


Figure 1 Multi-beam beam topologies for frequency re-use.

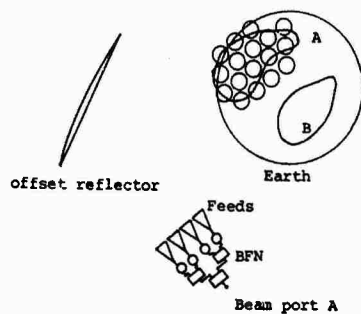


Figure 2 Contoured beam from offset reflector with feed array and BFN

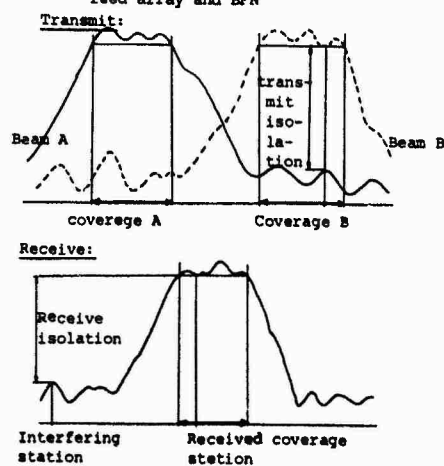


Figure 3 Microwave multi-beam lens antennas

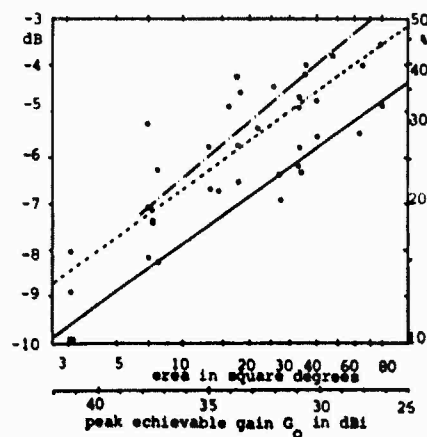
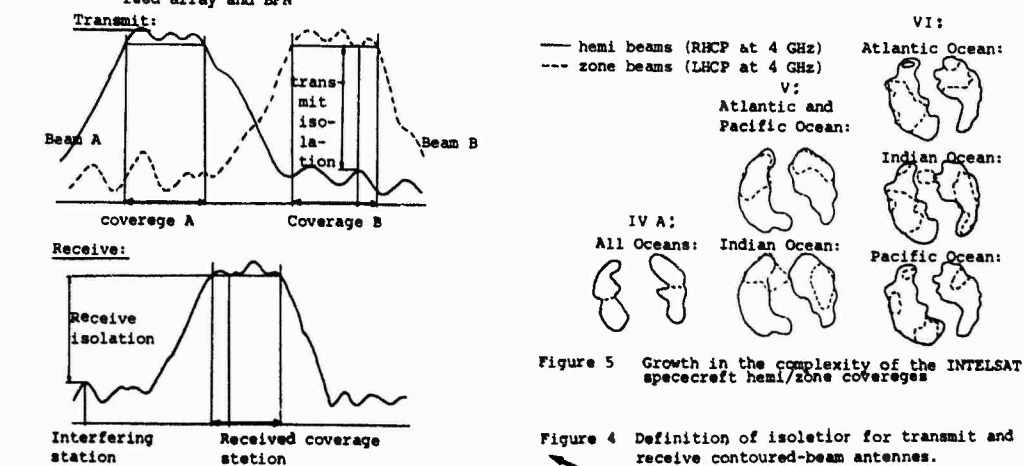


Figure 6 Contoured-beam efficiencies versus coverage area size for 3.2 m reflector at 4 GHz. The lines are determined by linear regression for the following three cases:

- INTELSAT VI antenna (ee)
- similar but with 1.3λ feeds (ee)
- .-. similar but with 1.3 feeds and relaxed coverage area spec. (oo)

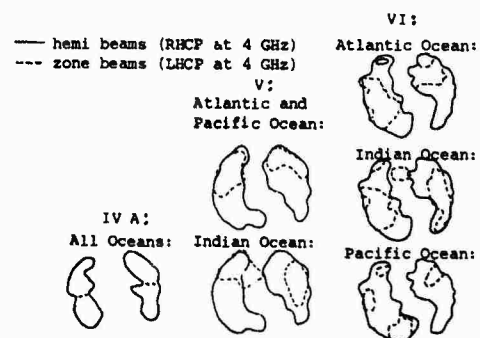


Figure 5 Growth in the complexity of the INTELSAT spacecraft hemi/zone coverages

Figure 4 Definition of isolator for transmit and receive contoured-beam antennas.

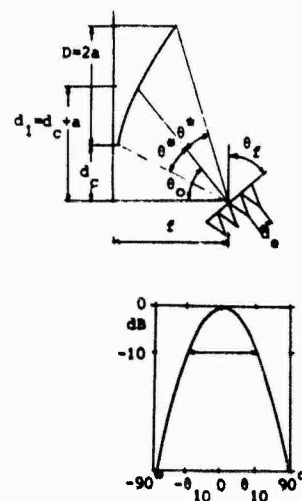


Figure 7 Reflector and feed parameters used in the simple analytic beam model.

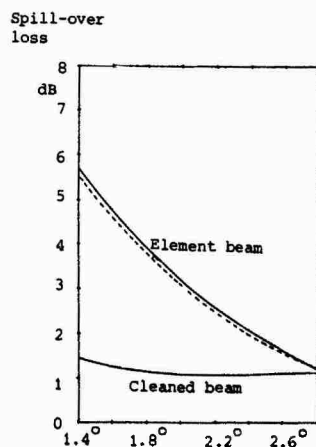


Figure 8 Spillover loss vs. spacing of component beams.  
 — simple analytic beam model  
 --- reflector antenna analysis program

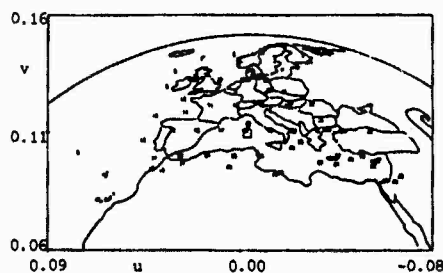


Figure 10 Examples of discrete stations (or sampling points) representing coverage-area specifications.

Figure 11 Contour plots for best analytical beam configuration

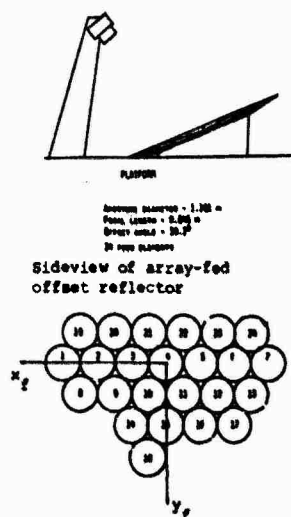


Figure 12 Initial RCS contoured beam reflector and feed array

Figure 13 FO/CTD element beams and contoured beam at 10.7 GHz

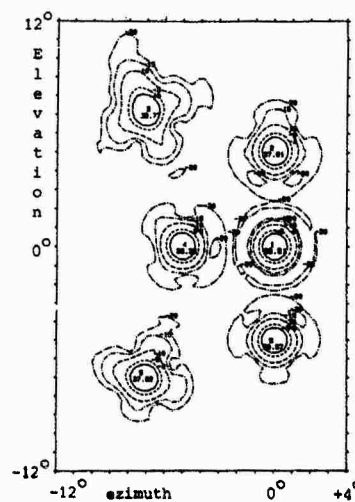
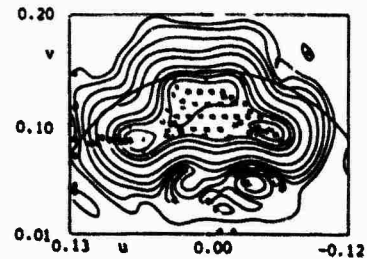
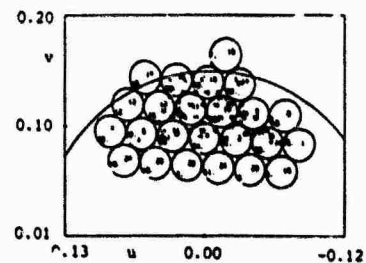
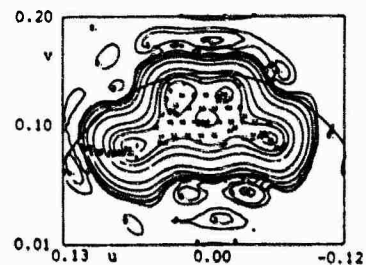
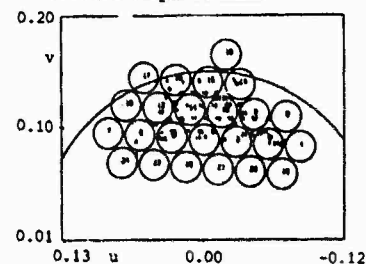


Figure 9 Gain contours 1, 3, 5, 10, 15 and 20 dB below peak gain level for on-axis and scanned element beams in a  $43\lambda$  diameter offset-fed paraboloid.



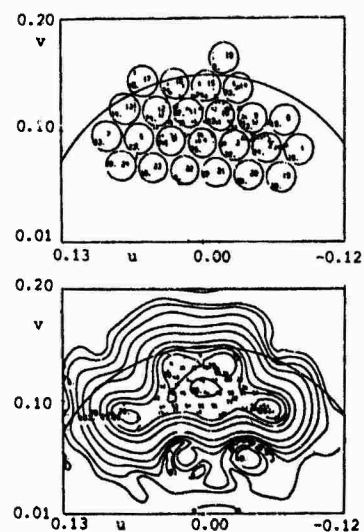


Figure 14 PO/GTD element beams and contoured beam at 11.7 GHz

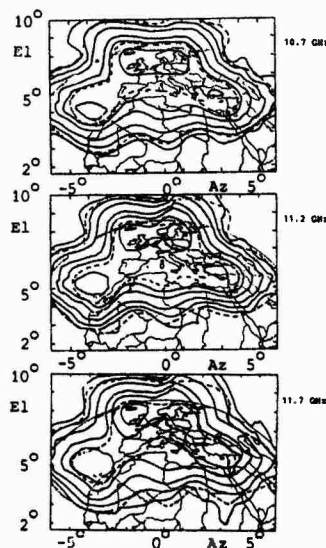


Figure 15 Measured gain contours 1, 3, 5, 10, 15 and 20 dB and calculated gain contours 3 and 20 dB below peak gain for ECS coverage array-fed contoured-beam reflector antenna  
— measurements (ERA)  
---- calculation

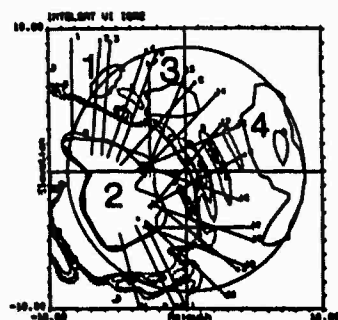


Figure 16 Nominal I-6 IOR zone 2 contours

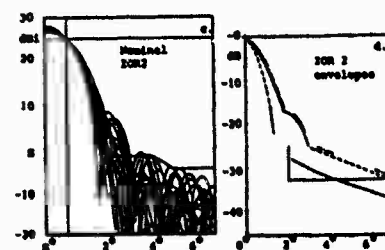


Figure 17 Gain roll-off patterns traces and envelopes of I-6 IOR zone 2.



Figure 18 IOR 2 synthesis stations

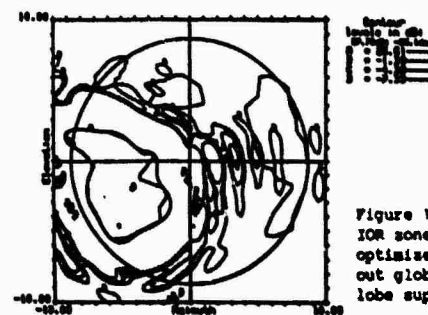


Figure 19 IOR zone 2 beam optimized without global side-lobe suppression

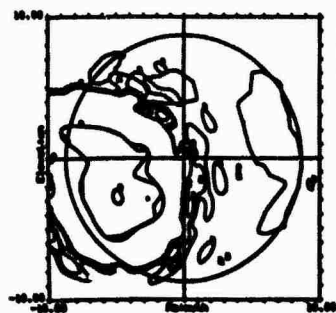


Figure 20 IOR zone 2 beam optimized with global side-lobe suppression

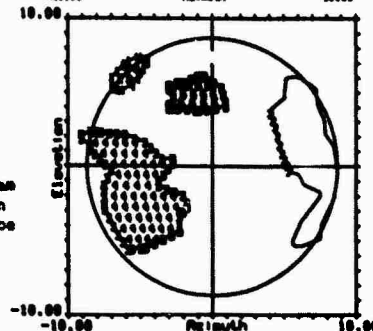


Figure 21 Synthesis stations for Indian Ocean region zone 2 beam

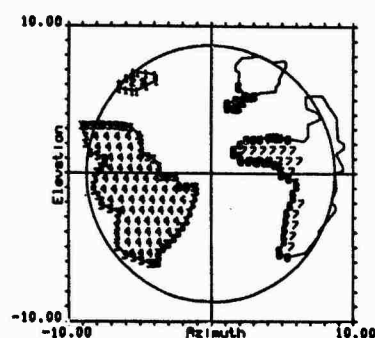


Figure 22 Synthesis stations for Atlantic Ocean region zone 2 beam

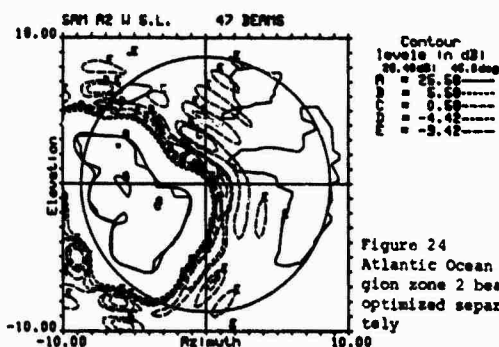


Figure 24 Atlantic Ocean region zone 2 beams optimized separately

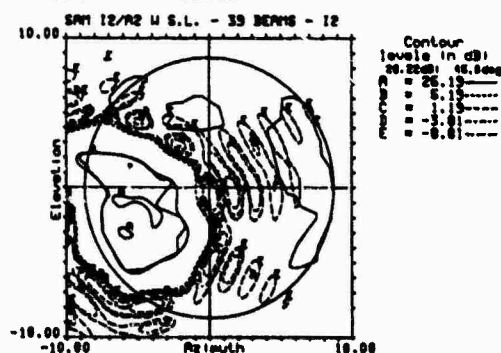


Figure 26 Indian Ocean region zone 2 beam optimized with 23 feeds common with Atlantic Ocean region zone beam

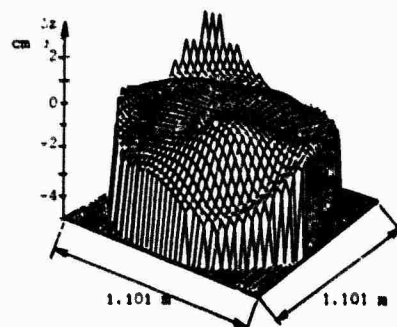


Figure 28 Deviations  $\Delta z$  of shaped reflector surface from paraboloid.

Figure 29 Isodirectivity curves for shaped reflector at edges of frequency band.

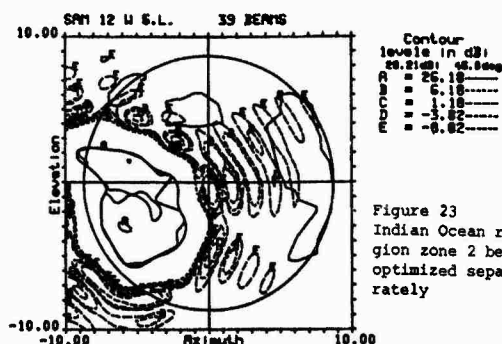


Figure 23 Indian Ocean region zone 2 beams optimized separately

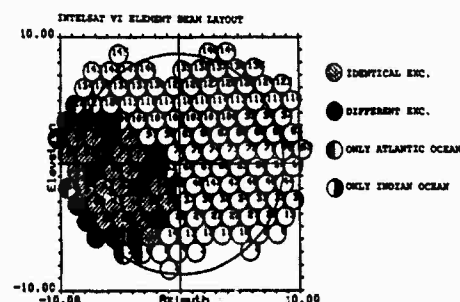


Figure 25 Shared and separate excitations for Indian and Atlantic Ocean region zone 2 beams

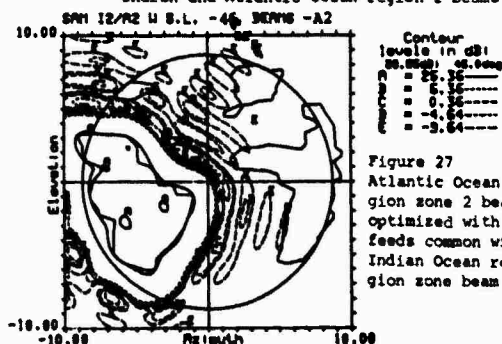
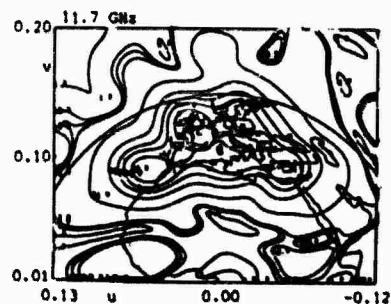
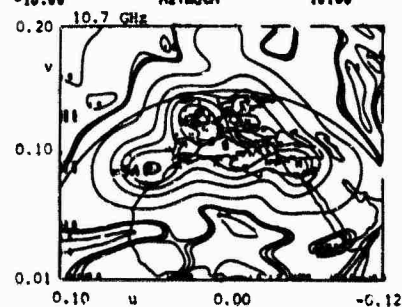


Figure 27 Atlantic Ocean region zone 2 beam optimized with 23 feeds common with Indian Ocean region zone beam





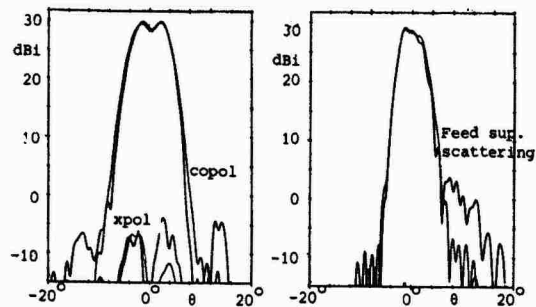


Figure 30 Measured and calculated principal pattern cuts at 11.7 GHz for shaped contoured-beam reflector.  
 — measurement (TUD)  
 ---- calculation

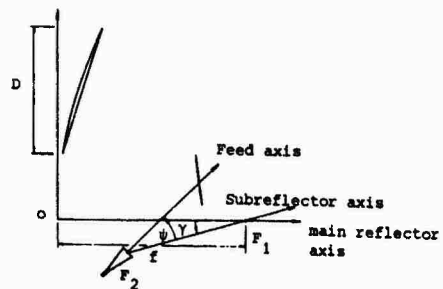


Figure 31 Conventional dual offset Cassegrain compensated for cross polarization and astigmatism

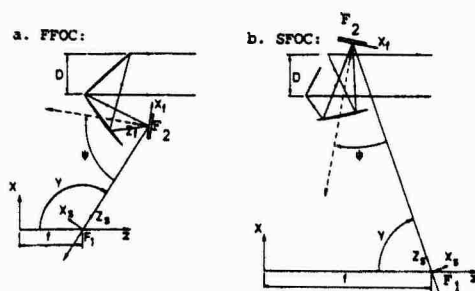


Figure 32 Alternative dual offset Cassegrain antennas compensated for cross polarization and astigmatism

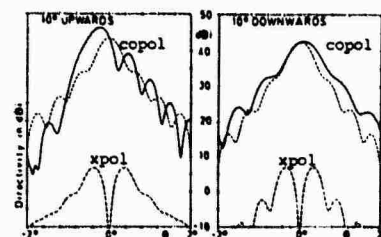
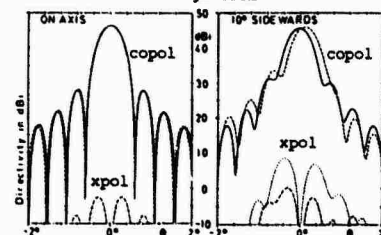


Figure 33 Principal pattern cuts for element beams in 120λ FFOC.

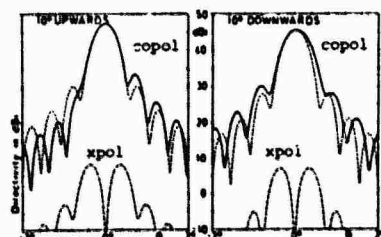
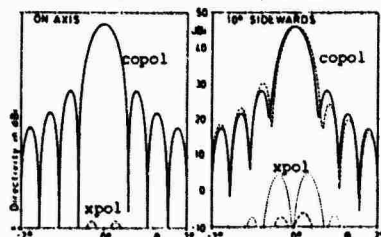


Figure 34 Principal pattern cuts for element beams for 120λ SFOC.  
 beam spacing

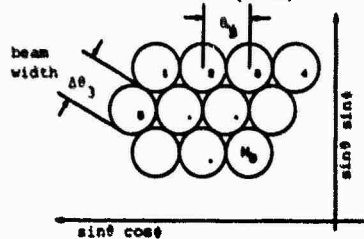


Figure 36 Array antenna element beams

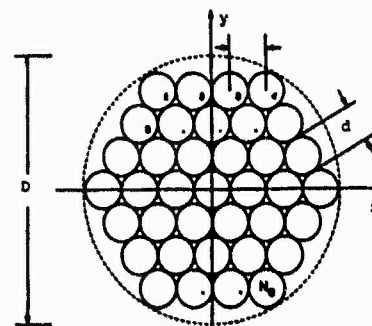


Figure 35 Array antenna element configuration

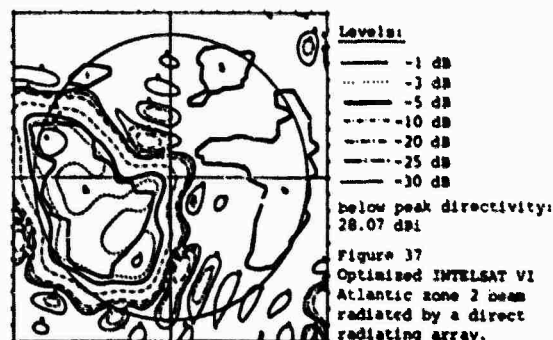


Figure 37 Optimized INTELSAT VI Atlantic zone 2 beam radiated by a direct radiating array.